Learning mathematics - letting the pupils have their say

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Learning mathematics – letting the pupils have their say

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Abstract

Pupil voice is an emerging force for change and improvement in many UK schools, but what is not fully understood is how best to access pupil voice within the specific context of secondary mathematics departments. This paper presents a research project designed to use pupils as co-researchers in increasing knowledge about how to improve learning in mathematics. Pupils within the school were selected and trained as "Ambassadors" to understand and disseminate innovative ways of learning mathematics into their school environment and to act to allow the voice of all the pupils in their year group to be heard. The project was intended both to raise the pupils' awareness of how learning mathematics could be different and to enable them to voice their newly-informed opinions about how best they learned mathematics. The pupils' current feelings about the way that they were taught mathematics were explored and but the focus of the project was on enabling the pupils to make informed decisions about how they felt their learning could be improved. The pupils' awareness of different ways of learning mathematics was raised by introducing them to alternative teaching approaches. The data generated was initially analysed by the pupils themselves in order to inform their teachers about their views and subsequently constant comparison analysis resulted in the outcomes reported here. The outcomes indicate that the students could have an important role in enabling schools to develop their teaching and improve their pupils' mathematical learning when that voice is both informed and authorised.

***Key words**: pupil voice; changing learning; Mathematical Resilience.*

Introduction

Pupil voice is increasingly becoming recognised as an important force for change in schools. This study considers an intervention designed to facilitate the consultation of pupils as a factor in one school's quest to improve further the learning and teaching of mathematics. The aim of this study was to investigate whether enlisting pupils in the school as co-researchers (Fielding 2001) would be a valuable and manageable way of enabling the pupils to articulate clearly how they considered mathematics teaching should be changed and improved in order for them to learn more effectively. In particular the study investigated how pupil voice could be informed and then utilised, within the context of one secondary school's mathematics department, as part of a drive to improve the learning of mathematical ideas and thus overall attainment. The school's intention in taking part in this study was to increase the overall attainment of the school's pupils at GCSE, an examination taken at age 16 in the UK and also to respond to an agenda, driven by government agencies, to increase the number of students studying STEM (science, technology, engineering, and mathematics) subjects at University (Roberts 2002).

The emphasis on co-researching (Fielding 2001; Flutter and Rudduck 2004) in this study affected both the research tools utilised and the analysis of the data generated. Previous studies have shown how important student perspectives can be to understanding the teaching-learning process; for example, Young-Loveridge et al. (2005) used the pupil's voice to establish the need to increase communication in mathematics, Jansen (2006) discovered what motivates students to take part in classroom discussions and Esmonde et al. (2009) showed how mathematics teaching can unintentionally reinforce existing inequities. However research using the pupils as co-researchers in setting out an agenda for change in mathematics teaching is not well represented in literature. Our work on Mathematical Resilience, a construct that suggests the efficacy of involving the learner in the whole process of learning mathematics, indicates that the pupils' voice would be an important pointer to how learning could be improved.

Quicke (2003) considered that there is much yet to learn about how pupils' views of learning may be encouraged to become "broader, [and] more reflective' (ibid., p. 51) Pupils' perceptions can be very different from, and very informative to, the adults who seek to help them learn but they cannot comment on what they do not yet know about (McIntyre et al. 2005). The intended outcome of the interventions were to scaffold the formation of a community of pupils cognisant of a variety of effective learning approaches in mathematics and to facilitate the expression of that community's voice on 'what works in our school' in terms of improving mathematical learning. This article characterises the struggle to enable pupils to form a broader, reflective and more informed view on how best they learn mathematics and to enable them to voice their views.

Theoretical Basis

The study was based in theoretical ideas on the "power for change of the pupils' voice" (Flutter 2007; Flutter and Rudduck 2004) and on ideas of building *mathematical resilience* (Johnston-Wilder and Lee 2010a and 2010b). If mathematics is difficult to master, as many people often say it is, then learners need to develop a positive adaptive stance to mathematics such that it will allow them to continue learning despite barriers and difficulties. This positive adaptive stance to mathematics has been termed elsewhere as mathematical resilience (Johnston-Wilder and Lee 2010a). The approaches used in this study were designed to exemplify ideas which had previously been shown to produce resilient behaviours in pupils. Such approaches have been shown to cause mathematical learning environments to be positive places where barriers to learning mathematics may be overcome. Pupils who have a high level of mathematical resilience know that it is worth persevering when faced with difficulties and know many ways of doing this, work collaboratively with their peers, have the language skills needed to express their understandings and have a growth theory of mathematical learning (Dweck 2000).

It is contended here that resilience is not only needed in learning mathematics; all learning requires resilience. However pupils require a particular resilience in order to learn mathematics because of various factors that include: the types of teaching often involved (Nardi and Steward 2003; Ofsted 2008), the nature of mathematics itself (Mason 1988; Jaworski 2010) and pervasive beliefs about mathematical ability being 'fixed' (Dweck 2000; Lee 2006). In this particular school the results discussed later also indicated that despite knowing about and using resilient learning behaviours in other curriculum areas, the pupils were not using the same behaviours when learning mathematics, either because they had become dissuaded from doing so or because they felt that such behaviours were inappropriate when learning mathematics.

By working with a group of students, termed "Ambassadors", the research team's intention was to include both teachers and pupils in the purpose of improving the mathematical learning within the school; thus the communicative aspects of resilient behaviours were particularly significant in this project. Vygotsky's socio-cultural theories concerning promoting dialogic interactions were important in the thinking and planning for the study; in particular his discussion of the potential of discourse to enable intra-mental ideas to subsequently become inter-mental (Vygotsky 1981 p. 162). The approaches used with the pupils, such as video making and peer teaching, arose in part from Vygotsky's work but also out of research such as that of Sfard (2001) Alexander (2008), Mercer and Littleton (2007) and Lee (2006) which confirmed that thinking and communicating are intricately intertwined, and that an increase in one is commensurate with an increase in the other.

Arguably, all studies that depend on the pupil's voice may be considered to also depend upon beliefs about self-efficacy (Bandura 1997). Self-efficacy can be taken to be "*an individual's belief in their capability to successfully complete an identified range of actions in a given field*" (Pampaka et al. 2011 p.171). In this study, two aspects of self-efficacy were especially salient. First, the notion that the pupils should see the result of voicing their thoughts in changes in the way that they learned, so that they could, in turn, see the worth given to their views. Second, the pupils' beliefs and values about their capability to learn mathematics were deliberately explored and often challenged in the workshop days. The ethos of these workshops was that mathematics lessons can be inherently interesting, involving and not "*boring and repetitive"* as one pupil described her current mathematics lessons. Mathematical learning was presented in ways that were accessible to all the participating pupils, promulgating the belief that all pupils can learn and become better at using and controlling mathematical ideas. A further belief, of the overwhelming importance of good marks or performativity widely held by teachers in the UK (Ball 2003), was challenged overtly and deliberately by supporting the contrasting belief that it was crucial to feel positive about learning mathematics, to work to build self-efficacy and resilience and for the pupils to feel assured about their ability to learn and improve and thus achieve in mathematics.

The pupil's voice is central to the study because of, "*the more simple and profound rationale of pupil voice which is that it affords teachers an opportunity to refocus their attention on what really matters—learners and how they learn best*" (Flutter 2007 p.345). Therefore, the project focused on learners and on facilitating an increase in their knowledge of which practices might work for them in the mathematics classroom. The project team, made up of teachers and university researchers, considered that co-opting the school's pupils as co-researchers over a sustained period may be an effective way of understanding where and what change might be needed, although it also meant that there was the potential for a re-balancing of power in the classroom (McIntyre et al. 2005). The pupils were construed as co-researchers in this project, involved in collecting data from their own experiences and those of their peers and in the analysis of that data. However, using Flutter and Rudduck's (2004) five rungs (0-4) of pupil participation, pupil participation in this research would rate at 3 (pupils as researchers), not quite at 4 (pupils as fully-active and co-researchers); although the pupils were involved in designing the research questionnaire and collecting and analysing data, they did not participate in setting out the research questions or in designing the research activity. Kirby (1999) and Clark et al. (2001) claim that involving young people as researchers generates high quality research data about the lives of young people, arguing that young people may be more open about their ideas and opinions with other pupils, making the data collected more trustworthy.

The Study

The school was an all-girls school in an urban setting in the Midland region of England. It is considered a 'high attaining school', that is, the results its pupils attain have so far placed it towards the top in English school league tables. Nonetheless, the school managers had identified a problem; the pupils' attainment overall in English language examinations was always higher than in mathematics. The mathematics AST (Advanced Skills Teacher), in conjunction with the school management and other members of the mathematics department, wanted to use 'pupil voice' to begin to narrow this gap. She had seen our work on Mathematical Resilience (Johnston-Wilder and Lee 2010a and 2010b) and invited us to join her department in this venture. The project focused on introducing a group of pupils and teachers to new-to-them strategies for engaging and empowering the pupils in their mathematical learning, building their understanding of mathematical resilience and using this group of individuals as conduits for change. It should be noted that use of pupil voice was already seen to be important to the school; they wanted to use pupil voice through '*insistent imperatives of accountability rather than enduring commitments to democratic agency*' (Fielding 2001 p.123). The school seemed to see consulting the pupils as 'a good thing' but had not necessarily thought about how they would respond to the outcomes of that consultation.

The university researchers came to the school as outsiders but had no wish to be used as '*outside experts to inform us'* (SooHoo 1993 p. 386), rather we intended to act to empower both the teachers who planned with us and the pupils themselves, as co-researchers, to inform the department about what would work best to enhance the learning of mathematics. All the planning and evaluation of the planning was done collaboratively by a research team that consisted of the AST, other teachers from the school and the university researchers. The programmes for the days in school were designed to enable the pupils to have an informed voice regarding 'what works' for their mathematical learning and to set up a mechanism that allowed their voice to be heard. The programmes allowed the pupils to experience how teaching could be different. Arguably by enabling the pupils to understand different ways of working, the pupil voice could become a 'powerful tool' (Flutter 2007; Flutter and Rudduck 2004) in helping teachers reflect on their practice in order to improve learning in mathematics. When the pupils' voice is authorised to comment (Cook-Sather 2002), pupils could be expected to react positively, making concrete and helpful suggestions.

Despite the school expecting the students to make helpful suggestions it remains the case that when pupils are asked about how their learning could be improved, pupils can give what may seem to be formulaic or naive answers. "*Far from saying 'Do something completely different', pupils tended very often to ask for more of teachers' existing or past practices or for extensions and elaborations of these*" (McIntyre et al. 2005 p.166). The research team believed that the pupils may not suggest different approaches to teaching, not because of the comfortable situation where pupils value the way that teachers teach, but rather the less comfortable situation where they have no experience of more effective ways to learn mathematics. We decided that pupils who have experienced different ways of learning mathematics may be in a better position to say 'what works' for them and planned the days accordingly.

The Work in School

Typically for a school in the English system, the school had 'set' or grouped its pupils into classes according to their attainment on internal examinations. The teachers chose two girls from each of these 12 classes or 'sets' to take part in the project as 'Ambassadors'. The girls were chosen because they had demonstrated a desire to take a lead within their own group and were willing to be part of this research. Thus the community of Ambassadors was formed from a mixture of girls in terms of mathematical attainment, mathematical confidence and ability to articulate their opinions. Articulate members of the community can have an overwhelming effect on any pupil voice consultation and those who may actually have more to say about their school experiences may find it harder to articulate their concerns (Arnot et al. 2004); consequently many different vehicles were designed to collect data so that every Ambassador's voice would be heard.

Three workshop days in school were planned over the spring and summer terms. These days were used in two ways: firstly to introduce the Ambassadors to different ways that mathematics can be learned and secondly to enable the Ambassadors to become co-researchers in discovering the opinions of their peers concerning learning mathematics. As co-researchers the Ambassadors also analysed data and evaluated the effectiveness of the new ways of learning that they experiences. They used two main ways to collect data: a questionnaire that was devised in conjunction with the Ambassadors (Appendix) and administered to all the pupils in Year Eight (12-13 years old) and journals in which they recorded their thoughts and feelings about learning mathematics. During the workshop days the university researchers introduced the Ambassadors to approaches to teaching that were known to increase learning in mathematics (for instance, Mercer and Littleton 2007; Lee 2006; Dweck 2000). Teachers were involved in the planning of these days and all teachers in the department were invited to participate in the sessions where these ideas were modelled with the Ambassadors. The Ambassadors were in turn invited to consider the approaches to which they were introduced and decide on which they felt were likely to be most advantageous in increasing their mathematical learning.

The Ambassadors experienced many ways of learning mathematics during the workshop. The tasks that the students were asked to undertake involved:

- using People Maths (Bloomfield and Vertes 2005). People Maths requires pupils to represent mathematical ideas using their own bodies, in this instance the pupils were asked to envision their shoulders and body as axes and to make straight line graphs using their arms and to solve a 'knot problem' made by linking hands by working together systematically;
- making a mathematics trail around their school by spotting mathematical ideas in the buildings and writing out a trail for other groups to follow;
- Using software to support learning mathematical concepts, for example using Grid Algebra (http://www.atm.org.uk/shop/products/sof071.html) and Autograph software http://www.autograph-maths.com/;
- making videos (see Johnston-Wilder and Lee (2010b) for a description of this in another context);
- creating a PowerPoint presentation about an aspect of mathematics of their choice that they found difficult;
- exploring where mathematics can be found in the real world and putting these images on a model chameleon;
- some drama role-play activities;
- data analysis of the data that they collected using the questionnaires.

The activities always involved a great deal of discussion and generally some element of choice in order to emphasise the self-efficacy of the pupils. For example, the pupils had to choose a topic for their video-making and they had to explore or play with the functionality of 'Autograph' in order to choose ways to make their screen resemble a given picture. Following each day a team of university researchers and mathematics teachers from the school met in order to review and evaluate the workshops and to plan following interventions. The team was also joined on each of these days by a drama teacher from Creative Partnerships (www.creative-partnerships.com) whose role was to inform about and model the use of drama in the service of mathematics learning. Drama can be seen as enabling dialogic communication and therefore working with this expert practitioner seemed to add to the expertise at the team's disposal for building the communicative aspects of the community that, as discussed above, was considered so important. We recognise here that the role of the drama specialist in the outcomes observed in this project needs further exploration but this is omitted for reasons of space and will be presented elsewhere.

The Data Sources

During the first of the days in school, a questionnaire was devised that was intended to examine how pupils currently felt about the way that the school encouraged them to learn mathematics. The university researchers drafted a questionnaire in order to present the Ambassadors with examples of questions which might be used, rejected or adapted. The questions in this draft questionnaire were derived from Dweck's work on fixed and incremental theories of learning (Dweck 2000) and on Fennema and Sherman's (1976) work on assessing attitudes to mathematics. The design parameters were to explore pupil's attitude to mathematics and the way that they viewed their ability to learn mathematics, especially if they displayed a growth or fixed (Dweck 2000) theory of learning mathematics. The questionnaire presented to the Ambassadors was an early iteration of a questionnaire that is in the process of being developed in collaboration with a colleague in the US (Kooken 2012, personal communication) in order to measure mathematical resilience, with a view to creating an instrument sufficiently sensitive to measure changes in this construct. The Ambassadors examined and discussed the draft questionnaire and made suggestions for changes to the questions. They proposed extra or alternative questions and rejected questions they felt were unnecessary or unhelpful. The Ambassadors attempted to both make the instrument more accessible to their peers and to ensure that it explored the ideas that they considered important. These submissions were recorded and changes were made to the draft questionnaire using the Ambassadors' suggestions. The questionnaire was used to provide data that the Ambassadors themselves could analyse during the workshops and therefore the data had to be meaningful for them. The questionnaire used is included as an Appendix.

The Ambassadors took the questionnaires to their normal mathematics classes and administered them to all the pupils in their classes; 267 questionnaires from a year group of 284 were collected. Hence the responses represent the feelings of almost all of the Year 8 girls in this cohort. The completed questionnaires were returned to the university team and the responses collated before the next workshop day in school. The responses were entered into a spreadsheet and a range of charts were created from the data. During the second day in school the Ambassadors perused the collated responses and graphical summaries and created PowerPoint presentations of what the data incated to them which were used in the final data analysis and also presented to the whole mathematics department in the school. The Ambassadors reported finding this both challenging and interesting; challenging in that they were asked to use complex mathematical ideas to analyse the outcomes and interesting because they found the data meaningful and they wanted to assemble it into a form that could report the reactions and feelings of their peers. Interestingly the results obtained from the questionnaire clearly showed that many of the pupils already had an intuitive appreciation of the power of working collaboratively and discussing their ideas when learning. However the questionnaires also indicated that in their experience these ways of working were rarely, if ever, used in the teaching of mathematics.

The Ambassadors were also asked on the first day to use journals to collect data on their feelings about, and reactions to, the mathematics they were learning and the way that they were learning mathematics in lessons and during our days with them. The mathematics teachers were consulted about this by the AST and agreement was gained from all the teachers. The Ambassadors were given dedicated books to use and were asked to record and express their feelings, positive and negative, towards the way that they were learning or not learning the mathematics they were engaged in. It was made clear that the Ambassadors should focus on their own and their peers' feelings and reactions and that the entries in the journals were not intended to criticise their teachers or record any negative personal comments but rather to focus on mathematical learning itself. The pupils wrote in their journals for about three months during the summer term of the academic year 2009/2010. The journals were brought to the second and third workshops, although two girls forgot to bring theirs on the second day. During these days the pupils discussed their entries between themselves and with the research team drawing attention to information in their journals which they considered to be important.

All 24 journals were collected immediately prior to the third day and a draft letter to the teachers was constructed by the research team from the information contained in the journals. This letter was discussed with the Ambassadors during the third workshop day and any changes requested and improvements decided upon were made to the document before it was sent to the teachers via the school members of the research team. The quotes given below either come directly from the pupils' journals, were recorded during the discussions of the letter or are from the final letter that the Ambassadors agreed was a reflection of what they wanted to say to their mathematics teachers. The data collection by our co-researchers was clearly focused on 'how they learn best.'

Table 1 – the data sources

Data Source	**Data collected by**	**Analysed by**
Questionnaire	Ambassadors from their peers	Researchers first collated the responses, the Ambassadors analysed and drew meanings from them
Journals	Ambassadors detailing their own reactions and feelings	Ambassadors discussed and shared entries that they considered important during workshop days. Full entries analysed by university researchers for this paper
Letter to teachers	Initial text created by researchers from entries in Ambassadors' Journals	Letter analysed and re-worded by Ambassadors
Pupils evaluations	Pupils written evaluations of the workshop days	University researchers and teachers in preparation for next workshop days and after the final workshop.
Field Notes of plans for work in school	University researchers	University researchers
Field Notes of discussions with teachers	University researchers	University researchers
Field notes of actions during workshop days	University researchers	University researchers

Analysis of the data

As a consequence of the design of the study, the data was largely provided by our co-researchers, the Ambassadors, who also assisted in the analysis of that data. The methodology used for analysis was complicated by the type of data generated and importance placed on involving the Ambassadors at each stage of the process. As previously mentioned the results from the questionnaires were entered into a spreadsheet prior to the workshop day and then were closely analysed by the Ambassadors who created presentations of the results from the questionnaires. Therefore in the final analysis both the original data from the questionnaires and the Ambassadors' analyses were considered.

The journals themselves were photocopied during the second and third days in school, so that the originals could be returned unmarked to the Ambassadors and the school, this data was later transcribed for analysis. The Ambassadors analysis generated during the discussion and sharing of their journals was carefully recorded in field notes. A preliminary analysis of the data in the journals and that from discussions with the Ambassadors on the second workshop day resulted in the construction of a draft 'letter to our teachers'. Involving students in the analysis of the data bolsters our claims that the results from this research are valid, in that the outcomes that are reported are the outcomes that the pupils themselves considered important. The role of the

university researchers was to make what the Ambassadors told us available to their teachers and to a wider audience.

The Ambassadors' views were made available to a wider audience by collating all the data and analysing it using a grounded theory approach (Glaser and Strauss 1967), returning repeatedly to the codes and the data until saturation was reached. Initially a series of open codes or themes were devised derived from the ideas raised by the Ambassadors in discussions and their PowerPoint presentations of the questionnaire data, predominantly amongst these were: collaborative working, teachers' actions, resilient stance, confidence and enjoyment. As the analysis proceeded these initial codes were extended, refined, modified or abandoned bearing in mind the initial question about enabling the pupils to voice their ideas about how they learned mathematics well and the necessity to clearly ground the outcomes in the data. The data was coded collaboratively by the university researchers and the analysis process continued refining the themes until agreement was reached on examples and non-examples for each theme. This process resulted in the themes discussed below, capturing the pupils' resilient approach to learning, the pupils view of effective teaching in mathematics and their view of themselves as learners. Involving pupils in analysis of the data was one of the ways of checking validity of the findings, in that the outcomes that are reported are the outcomes that the pupils themselves considered important.

The Outcomes from the analysis

Resilient approaches to learning can be well understood

It was evident from the data that the Ambassadors and their peers in the year group, knew intuitively about resilient ideas concerning effective teaching and learning of mathematics that are supported by research literature. For example, they knew that in the best lessons, teachers talk less and consequently pupils talk more, which echoes both Alexander's (2008) and Mercer and Littleton's (2007) findings. Many of the pupils' journals mentioned their mathematics teachers talking too much. *"When we are not involved enough we lose focus so we would like less teacher talk, more pupil work and more expectation of effort."* They recognised how important the use of language is and that they need to become proficient in the use of the mathematics register if they are to fully understand and be confident in using mathematical ideas. "*We would like teachers to give us more help on the meaning of words."*

Many of the attitudes to learning displayed by the girls in this high-achieving school corresponded to those that can be termed resilient. For example 78% said that they worked hard in mathematics lessons and 80% agreed with the idea that 'I can get smarter at maths if I work hard'. However, this means that about 20% of the girls reported that they did not work hard or did not consider that they would get smarter in maths through hard work. There was evidence that for some their otherwise resilient approach did not extend to mathematics; 94% of the girls reported that they were sure that they would be able to learn new work in all subjects but this level of confidence dropped by 6% (equivalent to 16 girls) when asked specifically about mathematics. Nevertheless 88% of the girls were confident in their ability to learn more mathematics. The resilient stance of the majority extended to their willingness to undertake tasks even if they knew that they might not 'do well' at the task; 17% said that they would not engage with such tasks and these are the pupils we feel would benefit from explicit promotion of resilience applied in mathematics.

Collaborative learning is known to be a resilient approach to learning mathematics (Swan 2006) and these pupils particularly valued working on mathematical ideas as part of a group. They indicated that they enjoyed both working with friends and working with people that they had not worked with before. They expressed a desire to support each other more in mathematics lessons. They asked for more opportunities for group activities, team work and co-operating with others. "*For example, one day this term, we did a GCSE problem and had to work as a group. It went well and everyone enjoyed it and began to work as a team."* Peers were also seen as being important to learning; the Ambassadors said that classmates should be allowed to help one another and they

recognised that they learn best when they are able to support each other and "*have a laugh occasionally"*.

The majority of the girls in Year 8 at this school clearly understood resilient approaches to learning mathematics. They knew that they need to 'do the talking' and to learn new words and ways of expression if necessary. They wanted to be involved in the process of learning as they knew this would help them to be successful learners. They wanted to support one another collaboratively when learning and were clear about the necessity to work hard in order to improve their learning in mathematics. This data so clearly echoes the literature on resilient learning (e.g. Hattie 2009; Stigler and Hiebert 2009; Mercer and Littleton 2007) that it seems likely that pupils in other schools will also be in the position to use such resilient strategies when learning mathematics if they are explicitly encouraged to do so.

Effective ways to learn mathematics

The majority of the ideas that were included in this theme came from the Ambassadors rather than the questionnaire data. The Ambassadors told us about teaching approaches that they considered would help them to become more effective learners of mathematics, most, but not all, arising from ways of working that they experienced in the workshops. They also gave examples of ways of teaching they found unhelpful.

Dynamic and involving learning activities: Notes from the days and the pupils' own evaluations showed that all 24 girls enjoyed making the videos: the particular elements that they mentioned about the days were the team work, being able to go outside, using ICT, the boost that the activities gave to their confidence and the fact that the activities were more interesting and fun than they had expected. One of the pupils who worked with Grid Algebra wrote in her evaluation of the day: '*something like nth term is usually boring but we understood it'.* After being asked to show the rest of the Ambassadors their work on Grid Algebra, one girl wrote '*I enjoyed making the presentation as I learned more about algebra. I would like to do something like this in my lessons as we could perform to each other and learn more'*. Another pupil wrote: *'we brought our confidence out, writing and really being creative'.*

The elements of choice the pupils were offered were considered important as were using visual aids and sharing work. A pupil wrote: *'all the projects were interesting and my thoughts about maths have really changed'*. The pupils told us that they enjoyed the more active ways of learning that they were offered in the workshop days and said they would like to do such activities more often in their mathematics lessons. More variety in their mathematics lessons would be appreciated by the pupils, less book work, more variety of tasks, fewer worksheets and more group work. The pupils did not believe that they learn or remember mathematical concepts met solely through bookwork. They would like more dynamic 'activities' such as presentations, independent work such as research, interaction with other people, projects or extended work.

The Ambassadors were convinced that collaborative and dynamic ways of working boosted pupils' confidence and motivated them to persevere in their learning. A further reason given in favour of such activities was that the pupils could help to support each other's learning when their main teacher was absent and they had to have an unfamiliar teacher. The pupils told us that they found it helpful when one of their teachers asked a pupil to take on the role of working at the board, either with a pre-prepared piece of teaching or by sharing their own working on a problem and giving the rest of the class opportunity to consider the pupil's reponse and ask questions about the ideas conveyed. They saw the value of working on more complex tasks, using a range of skills. Similarly, they would like more projects and extended work.

> *"We would like more interactivity, more games and interesting activities, more practical work and creative tasks, like making and testing helicopters as some did this term. We like more fun activities and we like adventures. Some of us enjoyed algebra puzzles and division with dominoes. Mathematics*

orienteering helped us to learn and have fun. We also suggest quizzes and mind-mapping– we can see it will help. We like maths we can recognise in the real world".

Many pupils, as in the quote above, seemed to see mathematics is 'a chameleon' discipline (Johnston-Wilder and Lee 2010a), that is the mathematics merges into the background of the 'real world' and cannot easily be seen. Only 76% of the pupils were sure that studying mathematics would help them to earn a living and 23% thought that studying mathematics might be a waste of time. For such pupils mathematics lessons that involve them in distinguishing how mathematics appears in the world in which they are interested or is useful to them or to their futures would be very helpful.

The pupils said that they like to learn using computers and they know that computers are not only for use when playing games. They felt that work such as making Powerpoint presentations helped them to learn and those who had the opportunity to use Grid Algebra recognised that it was a useful learning tool. However they said, *"Please can we have less MyMaths; we groan when we get MyMaths"*. MyMaths is an online resource widely used in English schools which presents strictly segmented or atomised explanations of mathematical topics and practice material that is assessed on-line, it is broadly similar to an on-line textbook, albeit an interactive one with embedded games. The pupils also suggested that they should sometimes be given the option of using ICT to support homework tasks.

The importance of the Teacher's stance: The journals show that the pupils consider that it is vital that they feel able to ask the teacher when they do not understand and that "*they explain and help if we are stuck*". They said that they like teachers to be sufficiently strict to ensure that pupils can learn in lessons, but not so severe that the teachers cannot be approached with questions and problems. The ethos of the class is crucial; they said that they needed a relaxed environment where they feel trusted and are allowed to talk to one another whilst working. The timing of the lessons was important; according to the pupils, lessons should be well paced and not involve, "*sitting still for too long and being bored*". The pupils are aware that they do not do well in an environment where the work is "*boring and repetitive*", a sentiment which resonates particularly strongly with Nardi and Steward's (2003) findings. The Ambassadors suggested that teachers should provide more accessible questions for people who are struggling and extension tasks for those who have understood. Sometimes, teachers could split the class into "*those who can do it and those who can't.*" They do not enjoy working in silence: "*we don't like the atmosphere of silence and it makes us feel locked in. We like it when people are talking, getting on with interesting work and able to ask questions with a helpful teacher."* They also told us that they don't like to be asked if they don't know – they feel "*dumb*."

Only 28% of the Year 8 girls enjoy mathematics all the time, although 55% enjoyed mathematics some of the time leaving 17% who did not enjoy it at all. Whilst it is to be expected that not all pupils will report enjoying mathematics all of the time, it is rather worrying that 17% reported not enjoying mathematics at all in the early years of their secondary school careers. However there are many messages from the Ambassadors that if taken account of may make mathematical learning more successful and thus enjoyable. Using a variety of dynamic and involving learning activities based in the real world, ensuring that there is less teacher talk and more pupil talk are not new ideas. The pupil voice may however lend power to the argument for teaching mathematics in this way.

Challenge, understanding and hard work

The Ambassadors' descriptions of 'good' lessons valued understanding and the pupils said that they liked lessons where all are given a chance to understand the essential elements. In their journals the pupils plead that teachers should "*Make sure all pupils understand the topic"*.

It was also clear that the pupils valued teachers who expect the pupils to do well; high expectations were emphatically appreciated by the pupils, "*we would like teachers to have higher expectations of us*". They felt that they would like more challenge and that they get more engaged when they are challenged. "*We don't mind hard work. We are not afraid to work hard."* The pupils enjoy working on difficult questions "*that will help us in the long run*."

This resilient stance was contradicted by the results from the questionnaire which asked if the pupils agreed with the statement "I sometimes would rather get good marks than understand the work." 40% agreed with this statement and a further 33% were not sure, leaving only 27% valuing understanding over good marks. This attitude cross matched well with the fact that 78% said that they preferred getting a good mark to being challenged. The results from the questionnaire indicate that the majority of pupils in the school are motivated by the idea of 'good marks' rather than the desire to understand and engage with mathematics for its own sake. This attitude is further emphasised by the 53% who disagreed with the statement "In addition to getting a right answer in maths, it is important to understand why the answer is correct." and the 58% agreed with the statement "It does not really matter whether you understand a mathematics problem if you can get the right answer."

Discussion

The most apparent conclusion was the extent to which the pupils' findings, journal entries and session feedback resonated with research about learning mathematics and our own research about how pupils become more mathematically resilient. The Ambassadors understood the importance of collaborative learning (Wiliam 2008; Mercer and Littleton 2007) and how important it is for the pupils to use the language of mathematics for themselves (Lee 2006; Sfard 2001). They also seemed to understand their role in learning, knowing that it was their hard work and perseverance when the work was challenging that would enable them to be successful, that is they understood the need for self-efficacy (Bandura 1997). The Ambassadors also displayed a predominantly incremental view of learning (Dweck 2000) during the workshop days, in that they knew that effort would result in success and that their understanding in mathematics could grow.

The pupils recognised the importance of variety in keeping them motivated and interested in their work (Hattie 2009; Stigler and Hiebert 2009). This is unsurprising perhaps, but nonetheless these pupils had to plead for collaboration, discussion and variety in their mathematics classes even in this well respected school. The majority of the Ambassadors clearly valued the way that learning mathematics was modelled during the days spent together. Most of pupils willingly presented their views about the way that they thought best helped them to learn mathematics both verbally during the workshop days and in writing in their journals. Those few who were less willing told us that they did not expect to be listened to and hence considered the process a waste of time. They became more willing to be involved as it became clear that they were viewed as co-researchers and that the data that they collected was recorded and considered by the group.
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The Year 8 Ambassadors' views conformed to the way that research defines 'effective' teaching: active, reflective, collaborative and grounded in the real world (Hattie 2009). Many of the pupils said very firmly that they enjoyed being challenged, and working on complex, but tractable problems. They are happy to "*work hard"* and for their teachers to have "*more expectation of effort"* from them. However they are adamant that their questions must be fully answered and all pupils' understanding valued and worked for. Lessons that involve the pupils in the process of learning, through choice of task, through collaborative learning and through active engagement are known to build resilience and increase the pupils' self-efficacy, a measure known (e.g. Kleanthous and Williams 2011) to be indicative of whether or not a pupil will continue to study mathematics once it is no longer compulsory for them to do so. The Ambassadors' preferred ways of learning are far from the atomised practice of mathematics teaching that is prevalent in many schools in

England (Nardi and Steward 2003). The pupils indicate that their mathematical learning will be enhanced if:

- their teachers move from total control of what goes on in the mathematics classroom and practices that tend to be repetitive and focussed on techniques, to giving choice and some autonomy to the pupils, and working with the pupils to develop mathematical understanding;
- the pupils are asked to collaborate, discuss and argue, that is to use discourse to think and learn about mathematics; and
- the pupils are given the opportunity to be active, resilient participants in the learning process.

There was some contradiction between the results from the questionnaires and the results from the journals concerning the importance of understanding. In the questionnaire only 27% unequivocally valued understanding over getting good marks whereas in the journals there is a clear plea for the teachers to "*Make sure all pupils understand the topic".* It may be that the pupils appreciate that when they understand their work it is likely that they will get good marks in examinations so that the one goes with the other. It is more likely, from the questionnaire results, that the dominant discourse in school, resonating with the performativity agenda, values marks above everything, including understanding, and therefore the quick answers given to a questionnaire reflect this discourse. However, the thoughtful and reflective data echoing the pupils' experiences in lessons and recorded in journals, presents a different argument and indicates that their pleas about understanding, challenge and hard work should be listened to if attainment in mathematics is to be improved.

The teachers themselves, who at the start of the process had all been willing to take part, became divided along a continuum by the second workshop. Those who were willing to listen to the pupils and learn from their experiences participated in the pupil workshops and saw what was going on. These teachers discussed the ideas with us and one of them invited their pupils to 'act as teacher' to demonstrate the ideas. However there were also others who came to share lunch with us but were reluctant to talk and did not let their pupils write in their journals during their lessons. Other mathematics teachers varied between these two extremes. It seemed that some of the teachers may have been made uncomfortable by what the pupils may say as "*Sustained and significant responding to pupil suggestions about what should happen in classrooms involves some change in the balance of classroom power"* (McIntyre et al. 2005 p.167). The teachers in this school are successful teachers according to many measures, the pupils offered ideas and some reasons for change and possibly the school's examination results will improve if the teachers use this project as "*an opportunity to refocus their attention on what really matters*" (Flutter 2007 p.345).

It seems likely that, before starting this work, pupils' knowledge and understanding of ways that would help them to learn mathematics effectively was present, partial but unarticulated. If this is correct, then it seems that allowing pupils to experience different ways of learning mathematics, reflect on their learning and be encouraged to use their voice to express the outcomes of their reflection, could have an important role in promoting their own awareness of their own knowledge of how to learn mathematics. As a research team, we demonstrated that explicitly listening to the pupils' expressed ideas and views was essential in giving authority to the pupils' utterances. However, without experiencing different ways of learning, the pupils would only be able to think in terms of "*extensions and elaborations"* (McIntyre et al 2005 p.166) of existing practices. The pupils' ideas were shown to be considered of value as they were both listened to and were responded to with changed practices. It is important to note the potential for damage from teachers who cease listening. At the beginning of the project some of the pupils were unwilling to engage because they did not expect to be listened to, 'they won't listen, we won't bother'. However, as these pupils were listened to, their engagement grew. Thus, in this sense the exercise of pupil voice in this project might be considered both empowering and powerful.

There is a current body of opinion in English schools that gives import to the pupil voice (Ofsted 2009 and NCSL 2007). Therefore many schools are currently taking steps to ensure that they have consulted their pupils. However, as we have seen, consulting the pupils and actually listening to

them are very different things, and if the pupils have no vision of how things could be different it is likely that they would have less to say. The value in consulting pupils in this way seemed to us to be that these pupils understood that how learning in mathematics could be different and thus they were able to give informed and reflective opinions on how they felt that they would learn best. Also, arguably more importantly, they were able to comprehend better that their own understanding of effective ways of learning, garnered from experiences in other contexts, continue to be effective when it comes to learning mathematics.

Conclusion

Recruiting students as co-researchers (Fielding 2001; Flutter and Rudduck 2004) proved significant in constructing ideas about how pupils feel they could learn mathematics effectively. The way that the project was construed enabled students to say '*do something completely different*' (McIntyre et al. 2005 p.166) because they had experienced different ways of learning and were authorised to evaluate those approaches. The "*power for change of the pupils' voice*" (Flutter 2007; Flutter and Rudduck 2004) was enhanced in ways that we found surprising; they knew about the efficacy of resilient approaches to learning and had clear and positive messages about how their ability to learn mathematics could be improved. They wanted their teachers to have high expectations of them, to support them in attaining challenging goals and they understood how powerful collaborative learning could be. Their desire to use resilient approaches to enable them to learn mathematics was strong; however they felt their current experiences in mathematics classrooms frequently discouraged the use of such approaches.

Our co-researchers thought predominately in 'growth' (Dweck 2000) terms about learning mathematics in the environment offered by the workshop days. They felt a need to discuss ways of learning that they had experienced that they felt were not helpful such as the teachers talking too much, not allowing them to talk to one another in class or making them feel 'dumb'. However in each case negative learning was mentioned it was qualified by something that they did want their mathematics teachers to do, such as introduce variety into lessons and to expect them to work hard. Raising the pupils' understanding of different ways to learn mathematics and authorising them to give their opinion of the value of the ideas seemed to allow the pupils to form reflective and frank views on approaches to learning and to voice their position about the way they would learn mathematics more effectively.

Pupil voice has a vital part to play in the continuous improvement of teaching and learning in mathematics. The Ambassadors took their responsibility as co-researchers very seriously. They welcomed their role in 'trying out' different ideas for learning mathematics and were honest in their reactions to the ideas they were exposed to, emphasising the ones they valued and choosing not to talk about others. It is not possible to say from this one study what the outcomes of other studies will be, but in this school we found willing and able co-researchers who cared deeply about their mathematical learning and their learning environment. Much more research will be needed in order to know for certain, but this study indicates the likelihood that involving pupils as co-researchers and scaffolding their thinking by giving them alternative experiences and models of mathematical resilience, has the power to validate where and what changes are needed to enable optimal learning in mathematics.

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Appendix: Questionnaire

Thinking about Learning

We want to know what you think about learning in general and learning mathematics in particular. Please answer all the questions as honestly as you can, there are no right or wrong answers.

Please state what year you are in _

Please state whether you are a girl or a boy _

Questions about how you think about intelligence and learning in all your lessons

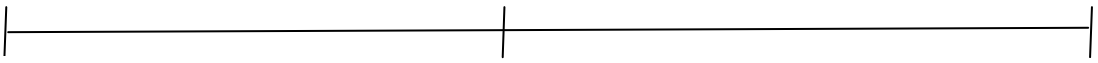
1. Think about ***all*** your experiences in school and answer this question

Tick the sentence that is most true for you

___ when I get new work in school, I'm usually sure that I will be able to learn it.

___ when I get new work in school, I often think I may not be able to learn it

Now show how true the statement is for you – indicate with an 'X' on the line



very true — true for me — sort of true

Read each sentence below and then circle the **one** number that shows how much you agree with it. Remember there are no right or wrong answers.

2. If I knew that I wasn't going to do well at a task I probably wouldn't do it even if I might learn from it.

1 strongly agree	2 agree	3 mostly agree	4 mostly disagree	5 disagree	6 strongly disagree

3. Although I hate to admit it I sometimes would rather get good marks than learn a lot.

1 strongly agree	2 agree	3 mostly agree	4 mostly disagree	5 disagree	6 strongly disagree

4. You can learn new things but you can't really change your basic intelligence

1 strongly agree	2 agree	3 mostly agree	4 mostly disagree	5 disagree	6 strongly disagree

5. If I had to choose between getting a good mark and being challenged in class I would choose … (Please circle one)

"good mark" "being challenged"

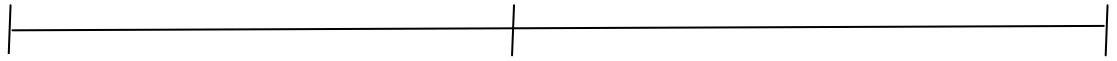
Thinking about how you learn in Maths

5. Tick the sentence that is most true for you

___ when I get new work in mathematics, I'm usually sure that I will be able to learn it.

___ when I get new work in mathematics, I often think I may not be able to learn it

Now show how true the statement is for you – indicate with an 'X' on the line



very true true for me sort of true

6. I can get smarter at maths if I work hard – please circle one

1 strongly agree	2 agree	3 mostly agree	4 mostly disagree	5 disagree	6 strongly disagree

Beliefs about learning mathematics

✓ Tick the sentence for each belief that reflects how you think – ***you can tick as many as you like*** - there are no right or wrong answers

Belief 1: About solving time-consuming mathematics problems- please choose one.

Maths problems that take a long time do not bother me.	
I feel I can do maths problems that take a long time to complete.	
I find I can do hard maths problems if I just persevere.	
If I cannot do a maths problem in a few minutes, I probably cannot do it at all.	
If I cannot solve a maths problem quickly, I stop trying.	
I am not very good at solving mathematics problems that take a while to figure out	

Belief 2: About understanding in maths - please choose one.

Time used to investigate why a solution to a maths problem works, is time well spent.	
A person who does not understand why an answer to a maths problem is correct, has not really grasped the problem.	
In addition to getting a right answer in maths, it is important to understand why the answer is correct.	
It is not important to understand why a mathematical procedure works as long as it gives the right answer.	
Getting a right answer in mathematics is more important than understanding why the answer works.	
It does not really matter if you understand a mathematics problem if you can get the right answer	

Belief 3: About how useful maths is - please choose one.

I study maths because I know how useful it is	
Knowing maths will help me earn a living	
Maths is a worthwhile and necessary subject	
Maths will not be useful to me in my life's work	
Maths is of no relevance to my life	
Studying maths is a waste of time	